Accurate Closed-Form Expressions for the Frequency-Dependent Line Parameters of On-Chip Interconnects on Lossy Silicon Substrate

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Abstract— **Accurate closed-form expressions for the frequency-dependent R, L, G, C line parameters of microstrip lines on lossy silicon substrate are presented. The closed-form expressions for the frequency-dependent series impedance parameters are obtained using a complex image method. The frequency-dependent shunt admittance parameters are expressed in closed form in terms of the shunt capacitances obtained in the low and high frequency limits. The proposed closed-form solutions are shown to be in good agreement with the electromagnetic solutions.**

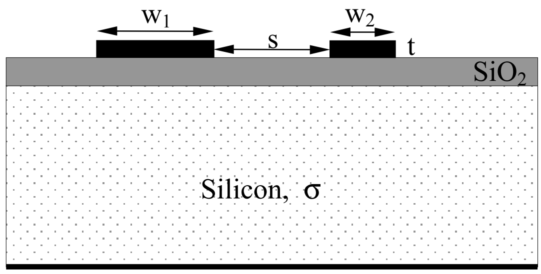

Fig. 1. Microstrip lines on lossy silicon substrate.

I. Introduction

There has been much interest in recent years in determining the frequency-dependent characteristics of on-chip interconnects and passive components on lossy silicon substrate. Transmission lines on silicon substrate have been studied extensively by full-wave electromagnetic analysis [1]-[4] and more recently by quasi-static EM approaches [4],[5]. Accurate closed-form expressions for the frequency-dependent transmission line characteristics that are suitable for computer-aided design, however, have not yet been reported. The development of closed-form expressions for the series $R(\omega)$ and $L(\omega)$ line parameters is especially difficult for silicon substrates with low bulk resistivity, such as in typical CMOS and BiCMOS processes, due to the excitation of frequency-dependent horizontal currents (eddy currents) in the conductive substrate.

A related problem arises, for example, in characterizing the effects of power lines above the conducting earth [6], [7] and in geophysics in studying magnetic variations associated with ionospheric or magnetospheric currents [8],[9]. Over the past several decades, complex image techniques have been developed to obtain approximate solutions that accurately describe the effects caused by the horizontal currents in the conducting earth.

In this paper accurate closed-form expressions for the impedance parameters of interconnects on conductive silicon substrate are developed using a complex image approach. It is shown that the complete frequency-dependent characteristics of the R, L, G, C line parameters can be formulated in terms of corresponding lossless configurations for which closed-form solutions are widely available. The new approach and the accuracy of the results is applied to a microstrip line on a lossy silicon substrate.

II. Modeling Approach

The new modeling approach is described for a microstrip line on a lossy silicon substrate with conductivity σ, as illustrated in Fig. 1. In the following the derivation of closed-form expressions will be given separately for the per-unit-length series impedance parameters $L(\omega)$ and $R(\omega)$ and shunt admittance parameters $C(\omega)$ and $G(\omega)$.

A. Series Impedance Parameters

For substrates with low resistivity, the time-varying magnetic fields in the substrate give rise to frequency-dependent horizontal currents (eddy currents) in the substrate. At higher frequencies the presence of these substrate currents can lead to a significant reduction in series inductance and substantial increase in series resistance or loss. For z-directed currents on the strip, the frequency-dependent complex inductance per-unit-length $L(\omega) + R(\omega)/(j\omega)$ may be determined by solving the quasi-static magnetic potential equation

$$\nabla^2 A_z(x, y) - j\omega\mu_0\sigma A_z(x, y) = 0 \qquad (1)$$

with the appropriate boundary conditions [4], [5]. Alternatively, by applying a complex image theory [9], the lossy substrate can approximately be represented in terms of an image plane located at a complex distance from the oxide/silicon interface, as illustrated in Fig. 2.

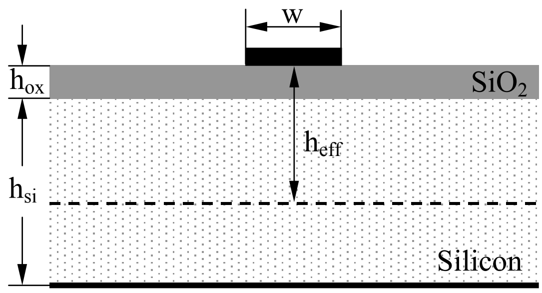

Fig. 2. Illustration of the image ground plane for a microstrip on a lossy silicon substrate.

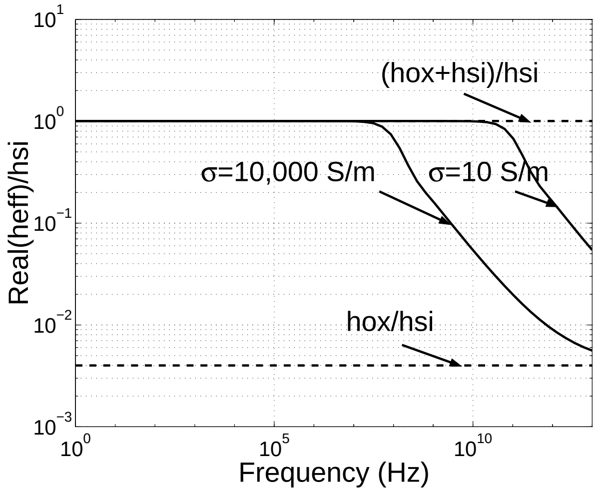

Fig. 3. Illustration of the variation in effective height as a function of frequency.

The resulting complex effective height of the microstrip is given by

$$h_{\mathrm{eff}} = h_{\mathrm{ox}} + \frac{1-j}{2}\delta \tanh[(1+j)h_{\mathrm{si}}/\delta] \qquad (2)$$

where h_{ox} is the oxide thickness, h_{si} is the thickness of the bulk silicon, and $\delta = 1/\sqrt{\pi f \mu_0 \sigma}$ is the skin depth of the bulk silicon. To illustrate the frequency-dependence of the effective height, Fig. 3 shows the real part of h_{eff} for a microstrip line with different substrate conductivities. Using the effective height, the series inductance and resistance parameters are determined as

$$L(\omega) = \mathrm{Re}\{L_{cf}(w/h_{\mathrm{eff}})\} \qquad (3)$$

and

$$R(\omega) = -\omega \mathrm{Im}\{L_{cf}(w/h_{\mathrm{eff}})\} \qquad (4)$$

where L_{cf} represents a closed-form expression for the external inductance per-unit-length of an ideal microstrip line given, for example, by [10]

$$L_{cf} = \frac{\mu_0}{4\pi} \ln\left\{1 + 32(\frac{h}{w})^2\left[1 + \sqrt{1 + (\frac{\pi}{8}\frac{w}{h})^2}\right]\right\} \qquad (5)$$

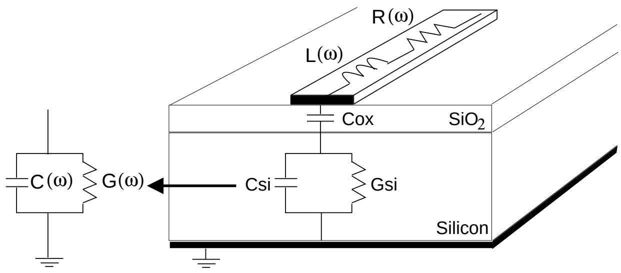

Fig. 4. Equivalent circuit topology for the shunt admittance of a microstrip on silicon.

B. Shunt Admittance Parameters

The frequency-dependent behavior of the shunt admittance can be represented in terms of the three element equivalent circuit shown in Fig. 4 [11]. To determine the frequency-dependent shunt admittance components $C(\omega)$ and $G(\omega)$, the electric behavior of the substrate in the low and high frequency limits is considered. At sufficiently low frequencies for which the relaxation frequency σ/ϵ of the substrate is much less than the operating frequency, the substrate behaves virtually like a perfect ground plane. The resulting shunt capacitance $C(\omega) = C_{\mathrm{ox}}$ is that of a microstrip of height h_{ox} and with $Si0_2$ as dielectric medium. The oxide capacitance C_{ox} can be determined using readily available closed-form expressions (e.g. [12], [13]). In the high frequency limit, the shunt conduction current in the lossy substrate can be neglected compared to the shunt displacement current. The resulting problem reduces to solving for the capacitance C_∞ of a lossless two-layer microstrip with capacitance for which closed-form expressions are available [14]. Using the equivalent circuit representation in Fig. 4, the silicon capacitance C_{si} is then obtained as

$$C_{\mathrm{si}} = \frac{C_{\mathrm{ox}}C_\infty}{C_{\mathrm{ox}} - C_\infty} \qquad (6)$$

The corresponding shunt conductance G_{si} in the silicon substrate is determined using the relaxation time constant $\epsilon_{\mathrm{si}}/\sigma_{\mathrm{si}}$ as

$$G_{\mathrm{si}} = \sigma_{\mathrm{si}}/\epsilon_{\mathrm{si}} C_{\mathrm{si}} \qquad (7)$$

With available closed-form expressions for C_{ox}, C_{si}, and G_{si}, the total shunt admittance components are given in closed-form by

$$G(\omega) = \frac{\omega^2 G_{\mathrm{si}} C_{\mathrm{ox}}^2}{G_{\mathrm{si}}^2 + \omega^2 (C_{\mathrm{si}} + C_{\mathrm{ox}})^2} \qquad (8)$$

and

$$C(\omega) = \frac{\omega^2 C_{\mathrm{si}} C_{\mathrm{ox}} (C_{\mathrm{si}} + C_{\mathrm{ox}}) + C_{\mathrm{ox}} G_{\mathrm{si}}^2}{G_{\mathrm{si}}^2 + \omega^2 (C_{\mathrm{si}} + C_{\mathrm{ox}})^2} \qquad (9)$$

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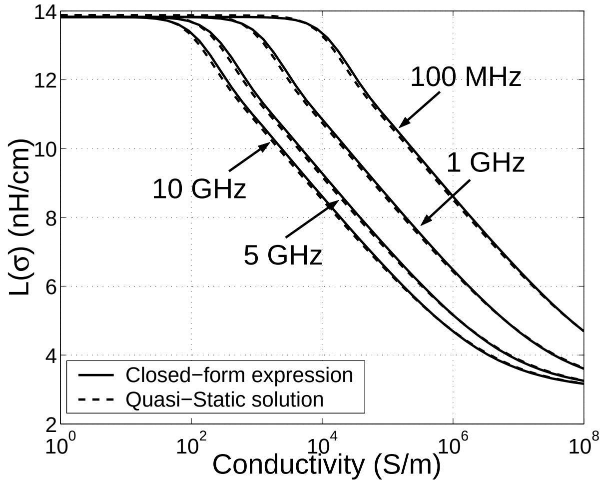
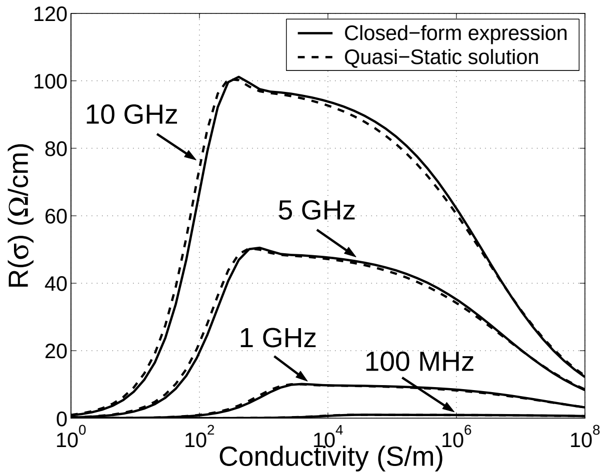
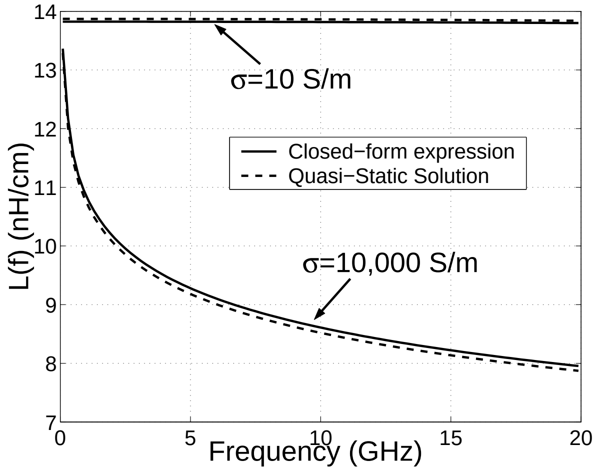


(a)

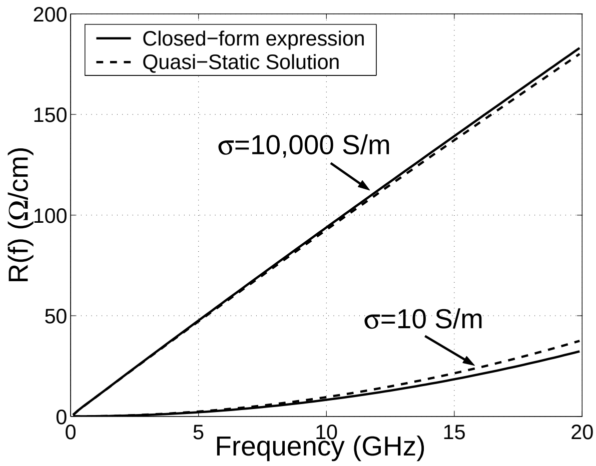


(b)

Fig. 5. (a) Distributed series inductance and (b) series resistance of a microstrip line on silicon substrate with $w = 4\mu\text{m}$, $h_{\text{ox}} = 2\mu\text{m}$, and $h_{\text{si}} = 500\mu\text{m}$ as a function of substrate conductivity.



(a)



(b)

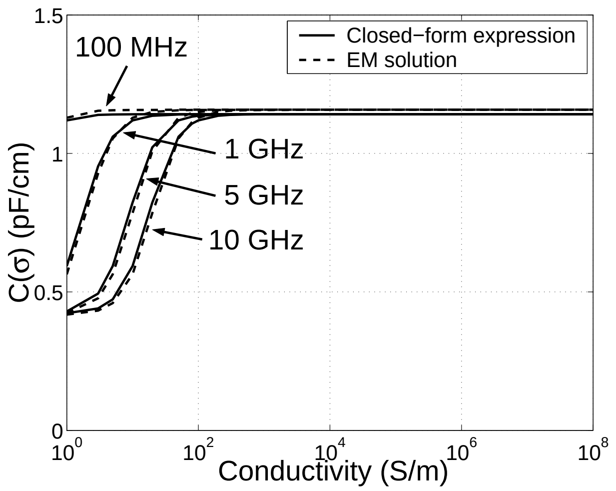
Fig. 6. Frequency dependence of (a) distributed series inductance and (b) series resistance of a microstrip line on silicon substrate with $w = 4\mu\text{m}$, $h_{\text{ox}} = 2\mu\text{m}$, and $h_{\text{si}} = 500\mu\text{m}$.

III. Results

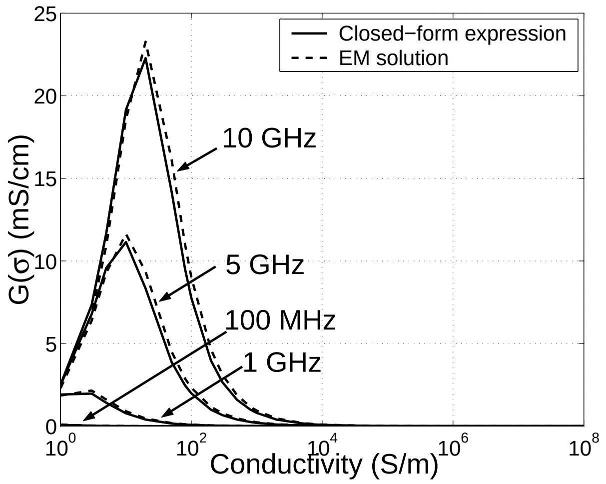
The proposed modeling methodology has been applied to various on-chip interconnects. To illustrate the accuracy of the method, the frequency-dependent R, L, G, C line parameters of a microstrip on silicon have been computed using the proposed closed-form expressions, and compared with the solution obtained by electromagnetic simulation.

Figures 5(a) and (b) show the frequency-dependent series inductance and series resistance parameters for a $4\mu\text{m}$ wide microstrip as a function of substrate conductivity. The EM solution was obtained by solving the magnetic potential equation (1) with a quasi-static solver [4], [5]. The difference between the closed-form solutions and the EM simulation results was found to be less than three percent over a wide range of substrate conductivities and frequencies. Figures 6(a) and (b) further demonstrate the significant frequency dependence in R and L for large substrate conductivities as well as the accuracy of the closed-form solutions.

The shunt admittance parameters of the microstrip on lossy silicon as a function of substrate conductivity are shown in Figs. 7(a) and (b). The EM solution was computed using a quasi-electrostatic spectral domain approach [15]. The results obtained using closed-form expressions are seen to be in good agreement with the EM solutions. The closed-form solutions are found to be within about five percent. It should be noted that, in general, the accuracy of the closed-form solutions for $C(\omega)$ and $G(\omega)$ directly depends on the accuracy of the available closed-form expressions for the per-unit-length capacitance of lossless single and multi-layer microstrip lines.



(a)



(b)

Fig. 7. (a) Distributed shunt capacitance and (b) shunt conductance of a microstrip line on silicon substrate with $w = 4\mu$m, $h_{\rm ox} = 2\mu$m, and $h_{\rm si} = 500\mu$m as a function of substrate conductivity.

IV. Conclusion

Accurate closed-form expressions have been developed to represent the complete frequency-dependent transmission line characteristics for microstrip lines on lossy silicon substrate. It was shown that the frequency-dependent R, L, G, C parameters can be accurately obtained in terms of closed-form solutions for lossless microstrip configurations using a complex image method for the series impedance parameters and low and high frequency limits for the shunt admittance parameters. The availability of accurate and fast closed-form expressions for the frequency-dependent transmission line parameters should be very useful for the design of silicon-based integrated circuits.

Acknowledgement

This research was supported in part by the NSF Center for the Design of Analog and Digital Integrated Circuits (CDADIC).

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